Period 4, March 18, 2025

Solve by the substitution method:

$$\begin{cases} x - y = 3 & \text{(The graph is a line.)} \\ (x-2)^2 + (y+3)^2 = 4. & \text{(The graph is a circle.)} \end{cases}$$

$x - \cancel{y} = 3$

$+\cancel{y} \quad +y$

$x = 3 + y$

$(3+y-2)^2 + (y+3)^2 = 4$

$(y+1)^2 + (y+3)^2 = 4$

$y^2 + 2y + 1 + y^2 + 6y + 9 = 4$

$2y^2 + 8y + 10 = \cancel{4}$

$-4 \quad -4$

$2y^2 + 8y + 6 = 0$

$2 \cdot 6 = 12$

$2 + 6 = 8$

$2y^2 + 2y + 6y + 6 = 0$

$2y(y+1) + 6(y+1) = 0$

$(y+1)(2y+6) = 0$

$y+1 = 0$ or $2y + 6 = 0$

$y = -1$ or $y = -3$

$x = 3 + y$	$x = 3 + y$
$x = 3 + -1$	$x = 3 - 3$
$x = 2$	$x = 0$
works $(2, -1)$	$(0, -3)$ works
True $2 - (-1) = 3$	$0 - (-3) = 3$ True
True $(2-2)^2 + (-1+3)^2 = 4$	$(0-2)^2 + (-3+3)^2 = 4$ True

Solve the system:

$$\begin{cases} 4x^2 + y^2 = 13 & \text{Equation 1} \\ [x^2 + y^2 = 10]\cdot -1 & \text{Equation 2} \end{cases}$$

$4x^2 + y^2 = 13$

$-x^2 - y^2 = -10$

$3x^2 + 0y^2 = 3$

$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{3}{3}$

$x^2 = 1$

$x = \pm 1$

$4(1)^2 + y^2 = 13$

$4 + y^2 = 13$

$y^2 = 9$

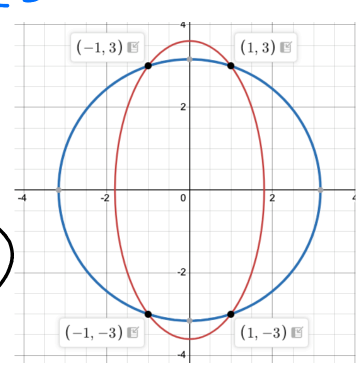
$y = \pm 3$

$(1, 3)$

$(1, -3)$

$(-1, 3)$

$(-1, -3)$



$y-3=x^2$

$$\begin{cases} y = x^2 + 3 & \text{Equation 1 (The graph is a parabola.)} \\ x^2 + y^2 = 9. & \text{Equation 2 (The graph is a circle.)} \end{cases}$$

$y-3+y^2=9$

$-9 \quad -9$

$y^2+1y-12=0$

$1\cdot -12=-12$

$-3+4=1$

$y^2-3y+4y-12=0$

$y(y-3)+4(y-3)=0$

$(y-3)(y+4)=0$

$y=3$ or $y=-4$

$(0,3)$ – only Solution

$3=0^2+3$ True

$0^2+3^2=9$ True

$y-3=x^2$

$3-3=x^2$

$0=x^2$

$0=x$

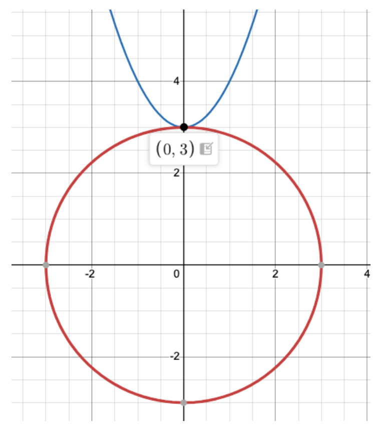
$y-3=x^2$

$-4-3=x^2$

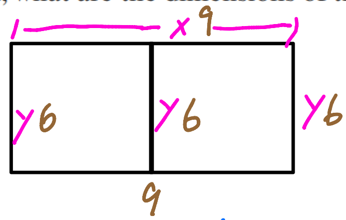
$-7=x^2$

$\pm i\sqrt{7}=x$

not gonna work



You have 36 yards of fencing to build the enclosure in **Figure 7.14**. Some of this fencing is to be used to build an internal divider. If you'd like to enclose 54 square yards, what are the dimensions of the enclosure?



Area $= xy = 54\ yd^2 \Rightarrow x = \boxed{\frac{54}{y}}$

$3y+2x=36$

$3y+2\left(\frac{54}{y}\right)=36$

$y\cdot\left(3y+\frac{108}{y}\right)=36\cdot y$

$3y^2+\frac{108y}{y}=36y$

$-36y \quad -36y$

$3y^2-36y+108=0$

$3(1y^2-12y+36)=0$

$1\cdot 36=36$

$-6\ -6$

$3(y^2-6y-6y+36)=0$

$3[y(y-6)-6(y-6)]=0$

$3(y-6)(y-6)=0$

$y=6$

$x=\frac{54}{y}=\frac{54}{6}=9$

works

The difference between the squares of two numbers is 5. Twice the square of the first number increased by the square of the second number is 22. Find the numbers.

$x^2 - y^2 = 5$ $\qquad$ $2x^2 + y^2 = 22$

$$x^2 - y^2 = 5$$
$$2x^2 + y^2 = 22$$
$$3x^2 + 0y^2 = 27$$
$$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{27}{3}$$
$$x^2 = 9$$
$$x = \pm 3$$

$$(\pm 3)^2 - y^2 = 5$$
$$\cancel{9} - y^2 = 5$$
$$\cancel{-9} \qquad -9$$
$$-y^2 = -4$$
$$y^2 = 4$$
$$y = \pm 2$$

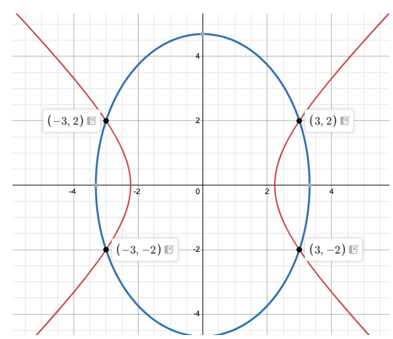
work

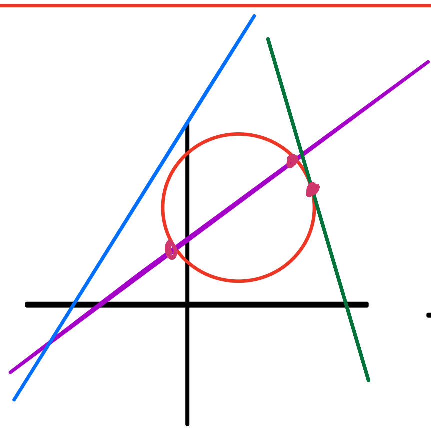
$(+3, 2)$

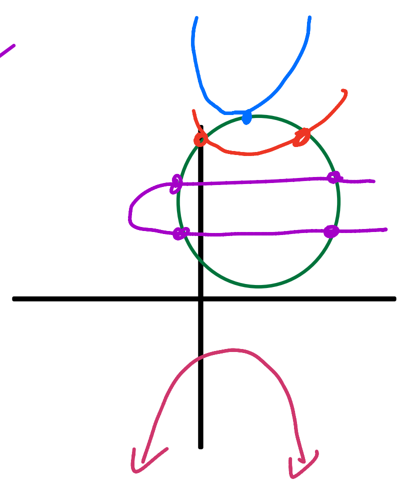
$(3, -2)$

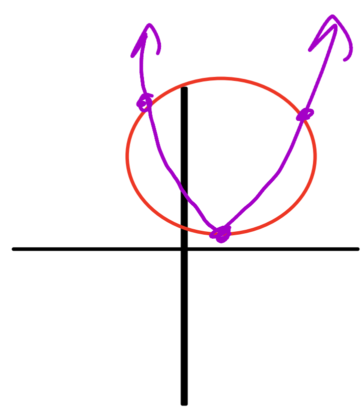
$(-3, 2)$

$(-3, -2)$









Solve the following equation in the complex number system. Express solutions in both polar and rectangular form.

$\sqrt[4]{x^4 + 4096i} = 0$

$x^4 = 4096i$ $\Rightarrow$ $x = \sqrt[4]{-4096i}$ $\Rightarrow$ $x = 0 - 8i$

$(0,-8)$

Write the solutions as complex numbers in polar form. Choose the correct answer below.

- A. $8(\cos 22.5^\circ + i\sin 22.5^\circ)$, $8(\cos 112.5^\circ + i\sin 112.5^\circ)$, $8(\cos 202.5^\circ + i\sin 202.5^\circ)$, $8(\cos 292.5^\circ + i\sin 292.5^\circ)$
- B. $8(\cos 67.5^\circ + i\sin 67.5^\circ)$, $8(\cos 157.5^\circ + i\sin 157.5^\circ)$, $8(\cos 247.5^\circ + i\sin 247.5^\circ)$, $8(\cos 337.5^\circ + i\sin 337.5^\circ)$
- C. $8(\cos 22.5^\circ + i\sin 22.5^\circ)$, $8(\cos 112.5^\circ + i\sin 112.5^\circ)$, $8(\cos 247.5^\circ + i\sin 247.5^\circ)$, $8(\cos 337.5^\circ + i\sin 337.5^\circ)$
- D. $8(\cos 67.5^\circ + i\sin 67.5^\circ)$, $8(\cos 202.5^\circ + i\sin 202.5^\circ)$, $8(\cos 247.5^\circ + i\sin 247.5^\circ)$, $8(\cos 292.5^\circ + i\sin 292.5^\circ)$

Write the solutions as complex numbers in rectangular form. Choose the correct answer below.

- A. $3.0615 + 7.3910i$, $-7.3910 + 3.0615i$, $-3.0615 - 7.3910i$, $7.3910 - 3.0615i$
- B. $-7.3910i$, $3.0615i$, $-3.0615 - 7.3910i$, $-7.3910 - 3.0615i$
- C. $-7.3910 + 3.0615i$, $-3.0615 - 7.3910i$, $-7.3910 - 3.0615i$, $3.0615 - 7.3910i$
- D. $7.3910 - 3.0615i$, $-7.3910i$, 3.0615, $3.0615 + 7.3910i$

DeMoivre's Theorem for Finding Complex Roots

Let $w = r(\cos\theta + i\sin\theta)$ be a complex number in polar form. If $w \neq 0$, w has n distinct complex nth roots given by the formula

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right] \quad \text{(radians)}$$

$$\text{or} \quad z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 360^\circ k}{n}\right) + i\sin\left(\frac{\theta + 360^\circ k}{n}\right)\right] \quad \text{(degrees)},$$

where $k = 0, 1, 2, \ldots, n-1$.

$\sqrt[4]{x^4 + 4096i}$

$\sqrt[4]{4096} = 8$

$$8\left(\cos\frac{270+360k}{4} + i\sin\frac{270+360k}{4}\right)$$

$$8\left(\cos\frac{270+360\cdot 0}{4} + i\sin\frac{270+360\cdot 0}{4}\right)$$

$8(\cos 67.5 + i\sin 67.5) = 8(0.3827 + i\,0.92388) = 3.06 + 7.39i$

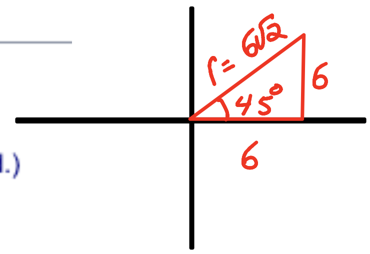
$$8\left(\cos\frac{270+360\cdot 1}{4} + i\sin\frac{270+360\cdot 1}{4}\right)$$

$8(\cos 157.5 + i\sin 157.5)$

$$8\left(\cos\frac{270+360\cdot 2}{4} + i\sin\frac{270+360\cdot 2}{4}\right)$$

$$8\left(\cos\frac{270+360\cdot 3}{4} + i\sin\frac{270+360\cdot 3}{4}\right)$$

Find all the complex fourth roots of $6+6i$. Write roots in rectangular form.



$z_0 =$ $1.7+0.3i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$z_1 =$ $-0.3+1.7i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$z_2 =$ $-1.7-0.3i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$z_3 =$ $0.3-1.7i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$$\sqrt[4]{6\sqrt{2}} = 1.7067$$

$$\sqrt[4]{6\sqrt{2}}\left(\cos\frac{45+360\cdot 0}{4} + i\sin\frac{45+360\cdot 0}{4}\right) = 1.7067(0.9808+i\,0.1951)$$

$$\sqrt[4]{6\sqrt{2}}\left(\cos\frac{\frac{\pi}{4}+2\pi\cdot 0}{4} + i\sin\frac{\frac{\pi}{4}+2\pi\cdot 0}{4}\right) = 1.7067(0.9808+i\,0.195)$$

$$1.6739 + .33297i$$

$$\sqrt[4]{6\sqrt{2}}\left(\cos\frac{\frac{\pi}{4}+2\pi\cdot 1}{4} + i\sin\frac{\frac{\pi}{4}+2\pi\cdot 1}{4}\right)$$

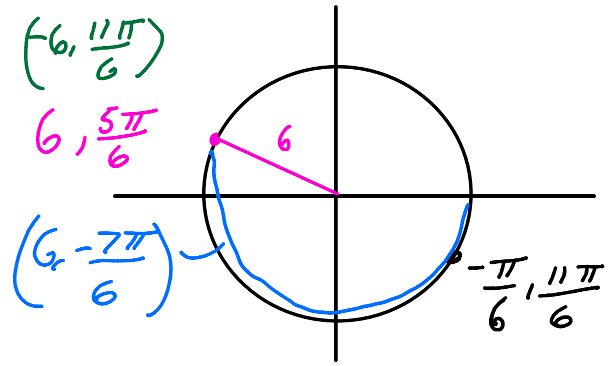
$$\sqrt[4]{6\sqrt{2}}\left(\cos\frac{\frac{\pi}{4}+2\pi\cdot 2}{4} + i\sin\frac{\frac{\pi}{4}+2\pi\cdot 2}{4}\right)$$

$$\sqrt[4]{6\sqrt{2}}\left(\cos\frac{\frac{\pi}{4}+2\pi\cdot 3}{4} + i\sin\frac{\frac{\pi}{4}+2\pi\cdot 3}{4}\right)$$

Select the representations that do not change the location of the point $\left(-6, -\frac{\pi}{6}\right)$.

Choose the correct answer below. Choose all that apply.

- [x] A. $\left(-6, \frac{11\pi}{6}\right)$
- [ ] B. $\left(6, \frac{\pi}{6}\right)$
- [ ] C. $\left(-6, \frac{17\pi}{6}\right)$
- [x] D. $\left(6, -\frac{7\pi}{6}\right)$



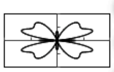
$r = \sin^2(4 \cdot 0) + \cos^2 4(0) + 0.3$

$1.3 = 0 + 1 + 0.3$

Use the polar mode of a graphing utility with angle measure in radians to graph the butterfly curve.

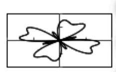
$r = \sin^2 4\theta + \cos 4\theta + 0.3$

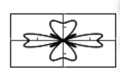
Choose the correct graph below.

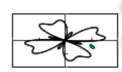
A. 

$\theta = 0$

$\theta = \frac{\pi}{2}$

B. 

C. 

D. 

All graphs [− 2,2] by [− 2,2] Xscl = 1 and Yscl = 1

2π

$r = \sin^2 4\left(\frac{\pi}{2}\right) + \cos^2 4\left(\frac{\pi}{2}\right) + 0.3$

$r = 0 + 1 + 0.3 = 1.3$

$r = \sin^2 \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + \cos^2 \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + 0.3$

$r = 0 + 1 - 0.3 = 0.7$